ECL 4340

POWER SYSTEMS

Lecture 8
Transmission Line Parameters

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Announcements

- For next two lectures read Chapters 4 & 5.
- HW #4 & Project #2, due on September 23, Friday
- Midterm Exam on September 29, Thursday

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Transmission Lines

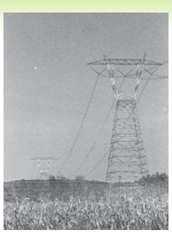

FIGURE 4.2

A 765-kV transmission line with self-supporting lattice steel towers (Courtesy of the American Electric Power Company)

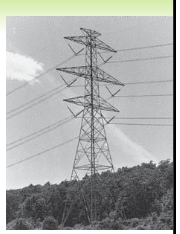

FIGURE 4.3

A 345-kV double-circuit transmission line with self-supporting lattice steel towers (Courtesy of NSTAR, formerly Boston Edison Company)

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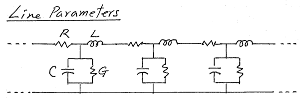
Transmission Line Parameters

Transmission Lines

Line Parameters



Distributed parameter representation

There are four line parameters, which are, in the order of importance,

- L: Series inductance H/m
- C: Shunt capacitance F/m
- R: Series resistance Ω/m
- G: Shunt conductance S/m

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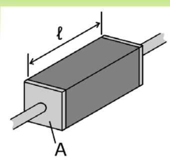
Transmission Line Parameters



1. Resistance R: depends on the length of the conductor ℓ, the resistivity of the material ρ (that increases with temperature), and inversely on the effective cross-sectional area A of the conductor:

$$R = \frac{\rho \ell}{A}$$

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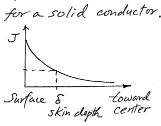
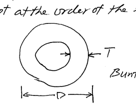
Transmission Line Parameters

The effective area A depends on the frequency due to the skin effect, where the current at 60-Hz frequency is not uniformly distributed throughout the cross-section; rather it crowds towards the periphery of the conductor with a higher current-density J for a solid conductor. Therefore, in ACSR conductors, the aluminum thickness T, is kept at the order of the skin depth.





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Transmission Line Parameters

Current-density J decreases exponentially such that at the skin-depth δ, the current density is a factor of e (= 2.718) smaller than that at the surface. The skin depth of a material at a frequency f is

$$\delta = \sqrt{\frac{2\rho}{(2\pi f)\mu}}$$

ρ: resistivity
μ: permeability

Ex. Aluminum conductor.

$\rho = 2.65\ \mu\Omega\text{-cm}$

$\mu = 4\pi \times 10^{-7}\ H/m$ (free space)

$\Rightarrow \delta = 18.75\ mm$

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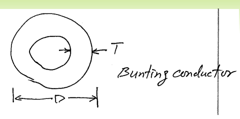
Transmission Line Parameters



In a Bunting conductor, $\frac{T}{D} = 0.3748$, $D = 1.302$ inches

$R_{dc} = 0.0787\ \Omega/\text{mile}$

$R_{bunting} = 0.0811\ \Omega/\text{mile}$

Both at 60 Hz, at the temperature of 25°C

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Transmission Line Parameters

Resistivity increases linearly with temperature:

$$\rho_{T_2} = \rho_{T_1}\left(\frac{T_2 + T}{T_1 + T}\right), \quad T: \text{temperature constant}$$

AC resistance or effective resistance:

$$R_{ac} = \frac{P_{Loss}}{|I|^2}\ \Omega$$

P_{Loss}: real power loss in W; I: rms current in A.

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Transmission Line Parameters

2. Conductance G:

In addition to the power loss due to I^2R in the series resistance, there is a small loss due to leakage current flowing through the insulator. This effect is amplified due to the corona effect where the surrounding air is ionized to conduct and a hissing sound can be heard in misty, foggy weather.

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Transmission Line Parameters

The corona problem can be averted by increasing the conductor size and by the use of conductor bundling. Conductance is usually neglected in power system studies because it is a very small component of the shunt admittance.

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Inductance: Solid Cylindrical Conductor

The inductance of a magnetic circuit that has a constant permeability μ can be obtained by determining the following:

1. Magnetic field intensity H, from Ampere's law
2. Magnetic flux density B ($B = \mu H$)
3. Flux linkages λ
4. Inductance from flux linkages per ampere ($L = \lambda/I$)

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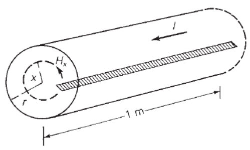
Inductance: Solid Cylindrical Conductor

Ampere's law:

$$\int H_{\tan} dl = I_{enclosed}$$ implies

$$H_x(2\pi x) = I_x \quad \text{for } x < r$$

implies



$$H_x = \frac{I_x}{2\pi x} \quad \text{A/m}$$

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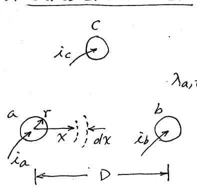
Transmission Line Parameters

3. Series Inductance L:



Per-phase inductance:

$$L_a \triangleq \frac{\lambda_{a,total}}{i_a} = \frac{\lambda_{a,i_a} + \lambda_{a,i_b} + \lambda_{a,i_c}}{i_a}$$

$\lambda_{a,total}$: superposition of flux due to i_a, i_b, i_c

Considering i_a alone, by Ampere's law at a distance x from conductor-a,

$$H_x = \frac{i_a}{2\pi x}, \quad B_x = \mu_0 H_x = \left(\frac{\mu_0}{2\pi x}\right) i_a$$

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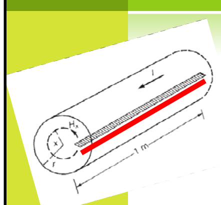
Transmission Line Parameters



Differential flux-linkage in a differential distance dx over a unit length along the conductor ($l=1$) is

$$d\lambda_{x,i_a} = B_x \cdot dx = \left(\frac{\mu_0}{2\pi x}\right) i_a \cdot dx$$

Assuming the current in each conductor to be at the surface (due to skin effect), integrating x from the conductor radius to infinity,

$$\lambda_{a,i_a} = \int_r^\infty d\lambda_{x,i_a} = \left(\frac{\mu_0}{2\pi}\right) i_a \int_r^\infty \frac{1}{x} dx = \left(\frac{\mu_0}{2\pi}\right) i_a \ln\frac{\infty}{r}$$

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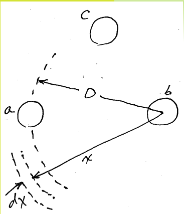
Transmission Line Parameters



Next, the mutual flux linking conductor-a due to i_b in conductor-b, noting that $D >> r$,

$$\lambda_{a,i_b} = \int_D^\infty d\lambda_{x,i_b} = \left(\frac{\mu_0}{2\pi}\right) i_b \ln\frac{\infty}{D}$$

Similarly, due to i_c,

$$\lambda_{a,i_c} = \left(\frac{\mu_0}{2\pi}\right) i_c \ln\frac{\infty}{D}$$

By superposition,

$$\lambda_{a,total} = \lambda_{a,i_a} + \lambda_{a,i_b} + \lambda_{a,i_c}$$
$$= \left(\frac{\mu_0}{2\pi}\right)\left[i_a \ln\frac{\infty}{r} + (i_b + i_c)\ln\frac{\infty}{D}\right]$$

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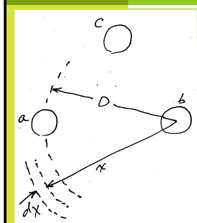
Transmission Line Parameters



By superposition,

$$\lambda_{a,total} = \lambda_{a,i_a} + \lambda_{a,i_b} + \lambda_{a,i_c}$$
$$= \left(\frac{\mu_0}{2\pi}\right)\left[i_a \ln\frac{\infty}{r} + (i_b + i_c)\ln\frac{\infty}{D}\right]$$

For a balanced three-phase transmission,

$$i_a + i_b + i_c = 0$$

$$\Rightarrow \lambda_{a,total} = \left(\frac{\mu_0}{2\pi}\right) i_a \ln\frac{D}{r}$$

$$\therefore L = \left(\frac{\mu_0}{2\pi}\right) \ln\frac{D}{r} \quad [H/m]$$

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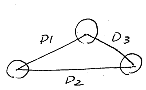
Transmission Line Parameters



For conductors at distances D_1, D_2, D_3 with respect to each other, define the equivalent distance D as the Geometric Mean Distance (GMD):

$$D = \sqrt[3]{D_1 D_2 D_3}$$

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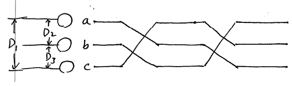
Transmission Line Parameters

for transposed lines.



In bundled conductor L is smaller:

by a factor of 0.7 for 3-conductor bundle w/ 18 inches spacing

by a factor of 0.8 for 2-conductor bundle

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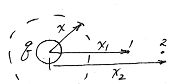
Transmission Line Parameters

4. Shunt Capacitance C:



Consider a charge q on a conductor, that results in dielectric flux lines and the electric field intensity E.

The dielectric flux-density D:

$$D = \frac{q}{(2\pi x)\times 1}, \quad E = \frac{D}{\varepsilon_0} = \frac{q}{(2\pi x)\varepsilon_0}$$

where $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, permittivity of air

The voltage of point 1 wrt point 2:

$$v_{12} = -\int_{x_2}^{x_1} E(x)\cdot dx = \left(\frac{q}{2\pi\varepsilon_0}\right) \ln \frac{x_2}{x_1}$$

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Parameters

The voltage of point 1 wrt point 2:

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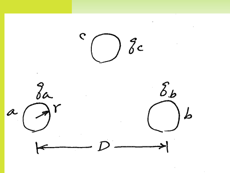

v_{ab} : voltage of a wrt b due to q_a, q_b, q_c

$$v_{ab,q_a} = \left(\frac{q_a}{2\pi\varepsilon_0}\right) \ln \frac{D}{r}$$

$$v_{ba,q_b} = \left(\frac{q_b}{2\pi\varepsilon_0}\right) \ln \frac{D}{r} = -v_{ab,q_b}$$

$$v_{ab,q_b} = -\left(\frac{q_b}{2\pi\varepsilon_0}\right) \ln \frac{D}{r}$$

For equidistance,

q_c does not produce any voltages between a & b.

$$\Rightarrow \quad v_{ab} = v_{ab,q_a} + v_{ab,q_b} = \frac{1}{2\pi\varepsilon_0}(q_a - q_b) \ln \frac{D}{r}$$

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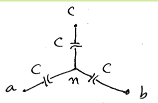
Transmission Line Parameters



Considering a hypothetical neutral point n, the capacitance C from each phase as shown,

$$v_{ab} = v_{an} - v_{bn}$$
$$= \frac{q_a}{C} - \frac{q_b}{C} = \frac{1}{C}(q_a - q_b)$$

Comparing the two eqns,

$$= \frac{1}{2\pi\varepsilon_0}(q_a - q_b)\ln\frac{D}{r}$$

$$C = \frac{2\pi\varepsilon_0}{\ln\frac{D}{r}} \quad [F/m]$$

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Transmission Line Parameters

For conductors at distances D_1, D_2, D_3, and transposed lines, the equivalent distance (GMD) is

$$D = \sqrt[3]{D_1 D_2 D_3}$$

In bundled conductor C is larger:

by a factor of 1.4 for 3-conductor bundle w/ 18 inches spacing

by a factor of 1.25 for 2-conductor bundle

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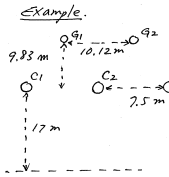
Transmission Line Parameters

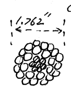
Example.



Tower: 3L3

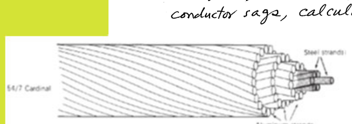
Conductors: Bluebird

Ground wires: 7/18" Utility Grade Steel



Aluminum Conductor Steel-Reinforced (ACSR)

Ignoring the effect of ground, ground wires and conductor sags, calculate ωL (Ω/km), ωC (μ℧/km)



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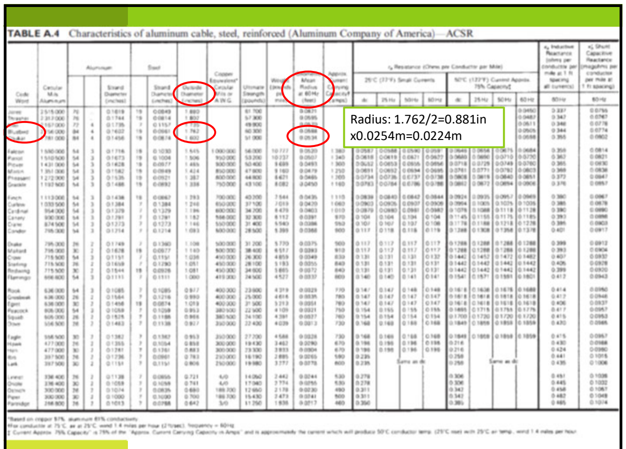

TABLE A.4 Characteristics of aluminum cable, steel, reinforced (Aluminum Company of America)—ACSR

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Transmission Line Parameters

Solution:

GMD: $D = \sqrt[3]{7.5 \times 7.5 \times 15} = 9.45\ m$

radius: $r = 0.0224\ m$

$$\omega L = (2\pi f)\left(\frac{\mu_0}{2\pi}\right) \ln \frac{D}{r}$$

$$= f \mu_0 \ln \frac{D}{r} = (60)(4\pi \times 10^{-7}) \ln \frac{9.45}{0.0224}$$

$$= 0.456\ \Omega/km$$

$$\omega C = (2\pi f) \frac{2\pi \varepsilon_0}{\ln \frac{D}{r}} = \frac{(2\pi)^2 (60)(8.85 \times 10^{-12})}{\ln \frac{9.45}{0.0224}}$$

$$= 3.467\ \mu\mho/km$$

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Transmission Line Parameters

Additional Notes

1. Internal Flux Linkage:

For accuracy, internal flux linkage can be added to compute inductance. In which case, the radius is modified as

$$r' = e^{-1/4} r = 0.7788\ r$$

For stranded conductors (ACSR), r' is replaced by GMR (geometric mean radius)

See A.4 Table

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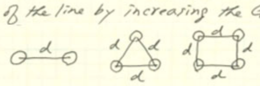
Transmission Line Parameters

2. Bundle Conductor:

It is common practice for EHV lines to use more than one conductor per phase, called bundling. Bundling reduces the electric field strength at the conductor surfaces, which in turn reduces or eliminate corona and its results: undesirable power loss, communications interference, and audible noie. It also reduces the series reactance of the line by increasing the GMR of the bundle.



$D_{SL} = \sqrt{D_s d}$ $\quad \sqrt[3]{D_s d^2}$ $\quad 1.091\sqrt[4]{D_s d^3}$

Inductance : $L_a = 2\times10^{-7} \ln \frac{D_{eq}}{D_{SL}}$ H/m

Capacitance : $\frac{2\pi\varepsilon}{\ln(D_{eq}/D_{SC})}$

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ACSR Table Data (Similar to Table A.4)

TABLE A8.1. BARE ALUMINUM CONDUCTORS, STEEL REINFORCED (ACSR) ELECTRICAL PROPERTIES OF MULTILAYER SIZES (*Cont'd*)

Code Word	Size (kcmil)	Stranding Al./St.	Number of Aluminum Layers	Resistance dc 20°C (Ohms/Mile)	Resistance ac-60 Hz 25°C (Ohms/Mile)	Resistance ac-60 Hz 50°C (Ohms/Mile)	Resistance ac-60 Hz 75°C (Ohms/Mile)	GMR (ft)	Phase-to-Neutral, 60 Hz Reactance at One ft Spacing: Inductive Ohms/Mile X_a	Phase-to-Neutral, 60 Hz Reactance at One ft Spacing: Capacitive Megohm-Miles X'_a
Flicker	477	24/7	2	0.1889	0.194	0.213	0.232	0.0283	0.432	0.0992
Hawk	477	26/7	2	0.1883	0.193	0.212	0.231	0.0290	0.430	0.0988
Hen	477	30/7	2	0.1869	0.191	0.210	0.229	0.0304	0.424	0.0980
Osprey	556.5	18/1	2	0.1629	0.168	0.184	0.200	0.0284	0.432	0.0981
Parakeet	556.5	24/7	2	0.1620	0.166	0.183	0.199	0.0306	0.423	0.0969
Dove	556.5	26/7	2	0.1613	0.166	0.182	0.198	0.0313	0.420	0.0965
Eagle	556.5	30/7	2	0.1602	0.164	0.180	0.196	0.0328	0.415	0.0957
Peacock	605	24/7	2	0.1490	0.153	0.168	0.183	0.0319	0.418	0.0957
Squab	605	26/7	2	0.1485	0.153	0.167	0.182	0.0327	0.415	0.0953

GMR is equivalent to r'

Inductance and Capacitance assume a D_m of 1 ft.

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ACSR Data, cont'd

$$X_L = 2\pi f L = 4\pi f \times 10^{-7} \ln \frac{D_m}{GMR} \times 1609 \ \Omega/\text{mile}$$

$$= 2.02\times10^{-3} f \left[\ln \frac{1}{GMR} + \ln D_m \right]$$

$$= 2.02\times10^{-3} f \ln \frac{1}{GMR} + 2.02\times10^{-3} f \ln D_m$$

Term from table assuming a one foot spacing

Term independent of conductor with D_m in feet.

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ACSR Data, cont'd

To use the phase to neutral capacitance from table

$$X_C = \frac{1}{2\pi f C} \quad \Omega\text{-m where } C = \frac{2\pi\varepsilon_0}{\ln\frac{D_m}{r}}$$

$$= \frac{1}{f}\times 1.779\times 10^6 \ln\frac{D_m}{r} \quad \Omega\text{-mile (table is in M}\Omega\text{-mile)}$$

$$= \frac{1}{f}\times 1.779\times \ln\frac{1}{r} + \frac{1}{f}\times 1.779\times \ln D_m \quad \text{M}\Omega\text{-mile}$$

Term from table assuming a one foot spacing

Term independent of conductor with D_m in feet.

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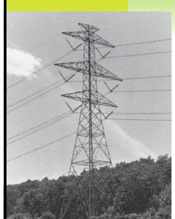
Additional Transmission Topics



- **Multi-circuit lines**: Multiple lines often share a common transmission right-of-way. This DOES cause mutual inductance and capacitance, but is often ignored in system analysis.
- **Cables:** There are about 3000 miles of underground ac cables in U.S. Cables are primarily used in urban areas. In a cable the conductors are tightly spaced, (< 1ft) with oil impregnated paper commonly used to provide insulation
  - inductance is lower
  - capacitance is higher, limiting cable length

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Additional Transmission Topics

- **Ground wires:** Transmission lines are usually protected from lightning strikes with a ground wire. This topmost wire (or wires) helps to attenuate the transient voltages/currents that arise during a lighting strike. The ground wire is typically grounded at each pole.
- **Corona discharge:** Due to high electric fields around lines, the air molecules become ionized. This causes a crackling sound and may cause the line to glow!

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Additional Transmission Topics

- **Shunt conductance:** Usually ignored. A small current may flow through contaminants on insulators.
- **DC Transmission:** Because of the large fixed cost necessary to convert ac to dc and then back to ac, dc transmission is only practical for several specialized applications
  - long distance overhead power transfer (> 400 miles)
  - long cable power transfer such as underwater
  - providing an asynchronous means of joining different power systems (such as the Eastern and Western grids).

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Tree Trimming: Before

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Tree Trimming: After

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